Training in Algebra and Analysis (TAA-2026)

Syllabus

Abstract algebra: Definition of a group, Cyclic groups, Dihedral groups, Symmetric groups, Matrix groups, Group homomorphisms and examples, Subgroups, Cosets, Conjugacy classes, Normal subgroups, Quotient groups, Lagrange's theorem, Isomorphism theorems. Actions of groups on sets, Symmetric group, Alternating group, Cayley's theorem. Direct products of groups, Group automorphisms, Sylow's theorem, Semi-direct products of groups, Classification of groups of small order, Classification of finite abelian groups, Composition series, Jordan-Holder theorem, Commutator subgroups, Solvable groups, Nilpotent groups.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions.

Linear algebra: Vector spaces, Subspaces, Linear dependence, Basis, Dimension, Algebra of linear transformations, Types of matrices and their properties, Algebra of matrices, Rank and determinant of the matrices, and system of linear equations, Characteristic polynomial, Eigenvalues and eigenvectors, Diagonalization, Minimal polynomial, and the Cayley-Hamilton theorem, Matrix representation of linear transformations. Change of basis, Canonical forms, Diagonal forms, Triangular forms, and Jordan forms. Inner product spaces, Orthogonal bases, and the Gram-Schmidt orthonormalization process, Bilinear and quadratic forms, and classification of quadratic forms.

Analysis: Elementary set theory, Countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, Supremum, Infimum, Sequences and series, Convergence, Power series (of one real variable), Radius and interval of convergence, Continuity, Uniform continuity, Differentiability, Intermediate value property, Rolle's Theorem, Mean value theorem, L'Hospital rule, Taylor's theorem, Taylor's series, Maxima and minima, Riemann sums and Riemann integral, Improper Integrals, Monotonic functions, Types of discontinuity, Functions of several variables, Directional derivative, Partial derivative, Maxima and minima, Saddle point, Method of Lagrange's multipliers, Derivative as a linear transformation, Inverse and implicit function theorems.